Monopoly Pricing with a Public Option*

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Abstract

This paper examines how a profit-maximizing monopolist competes against a free but capacity-constrained public option. The monopolist strategically restricts its supply beyond standard monopoly levels, thereby intensifying congestion at the public option and increasing consumers' willingness-to-pay for guaranteed access. Expanding the capacity of the public option always reduces producer welfare and, counterintuitively, may also reduce consumer welfare. In contrast, introducing a monopolist to a market served only by a capacity-constrained public option unambiguously improves consumer welfare. These findings have implications for mixed public-private markets, such as housing, education, and healthcare.

JEL Classifications: D42, D45, H42, L12

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1. Introduction

Government-run programs like housing, education, and healthcare provide a critical safety net by making essential goods and services accessible through free or heavily subsidized offerings. Prominent examples include subsidized public housing in Singapore, tuition-free universities in Norway and Germany, and free healthcare in United Kingdom, Canada, and Australia. These programs, however, often face demand that exceeds their supply, leading to rationing through waiting times or lotteries. As a result, profit-maximizing firms frequently emerge to provide the same goods and services, either as a stopgap or as an alternative to the rationed public provision. Thus, such settings are best understood as *mixed markets* where private providers operate alongside a capacity-constrained *public option*.

This paper focuses on the case in which private provision substitutes for the public option. For instance, individuals often choose between public and private education, but rarely combine the two. In such settings, public options serve not only as a safety net, but also as potential sources of competitive pressure on their private counterparts. This paper examines the strategic interactions that emerge when private providers compete against a rationed public option.

To that end, I present a parsimonious model of a mixed market in which the same good is supplied by a capacity-constrained public option and a profit-maximizing monopolist. There is a unit mass of risk-neutral consumers with heterogeneous valuations for the good. Each consumer has a unit demand and chooses to acquire the good from one of the two suppliers. The public option offers the good for free but randomly rations it whenever demand exceeds capacity. In contrast, the monopolist faces no capacity constraints and sells the good through a screening mechanism.

The screening problem in this setting differs from the standard monopoly problem in three key ways: (i) the monopolist cannot fully exclude buyers from obtaining the good due to the presence of the public option; (ii) each buyer's outside option is endogenously determined by her valuation for the public option and is therefore type-dependent; and (iii) by limiting its own supply, the monopolist increases the demand for the public option, which exacerbates rationing and worsens the buyers' outside option. As a result, the mechanism shapes not only the monopolist's allocation and pricing decision but also the broader mixed-market environment in which it operates.

The first result establishes that the optimal mechanism takes the form of a posted price: buyers whose valuation exceeds a certain threshold purchase from the monopolist, while the lower-value buyers rely exclusively on the public option. Crucially, this threshold ensures that the public option is rationed, which reflects the monopolist's ability to strategically exploit the public option's capacity constraint in order to raise buyers' willingness-to-pay for guaranteed access. The threshold type is therefore indifferent between receiving the good for free through a rationed allocation and purchasing it at the posted price, where the degree of rationing is itself determined by the threshold type.

Based on the characterization of the optimal mechanism and the resulting mixed-market outcome, the paper then analyzes how consumer and producer welfare vary with the public option's capacity. This comparative statics exercise is particularly relevant for policies aimed at introducing a public option into an initially monopolistic market, or expanding the capacity of an existing one within a mixed market. I first show that the monopolist responds to a capacity increase by further restricting its own supply in order to keep the public option congested. Consequently, the monopolist's market share and its profit decline as capacity grows.

The implications for consumers, however, are more nuanced. Absent any strategic response by the monopolist, a capacity increase would naturally ease rationing at the public option. But the monopolist offsets this benefit by raising the threshold for guaranteed access, which shifts even more consumers to the public option. Furthermore, as the threshold increases, the marginal buyer becomes a higher-type agent, potentially leading to an increase in her willingness-to-pay for guaranteed access.

I show that even with the monopolist's offsetting response, an increase in capacity always eases rationing, which improves the welfare of low-value buyers who rely on the public option. On the other hand, I show that an increase in capacity could, counterintuitively, lead the monopolist to raise its price. Hence, capacity-constrained public options may fail to exert competitive pressure. Nonetheless, I identify necessary and sufficient conditions on the buyers' type distribution under which a capacity expansion always lowers the monopoly price, benefiting even the high-value buyers who do not use the public option.

Finally, the paper considers the impact on consumer welfare when a monopolist enters a market initially served solely by a public option. I show that consumer welfare unambiguously improves: high-value buyers gain from the introduction of guaranteed access via the monopolist, while low-value buyers benefit from reduced congestion at the public option as some of its demand shifts towards the monopolist.

Of course, the tractable model developed in this paper does not capture all dimensions of mixed markets. In particular, it assumes that the public option provides the same quality good as the monopolist for free, and that the monopolist operates as a substitute to the public option. These assumptions are not only satisfied in some real-world settings, but they also represent the most favorable conditions under which a public option might exert competitive pressure. Yet, as the main comparative statics results reveal, even under these favorable conditions, introducing or expanding a public option does not necessarily improve consumer welfare.

In the Online Appendix, I consider the more general setting in which the public option supplies a good of lower quality at a subsidized price, characterize the optimal selling mechanism, and discuss the extent to which the qualitative results of the paper extend to the general setting. Additionally, I consider the case where the monopolist operates as a complement to the public option, with consumers "topping up" their demand in the private market. Unsurprisingly, the public option no longer exerts a competitive pressure on the private market in this setting, but a capacity expansion improves consumer surplus because all consumers now rely on the public option.

Related Literature: This paper relates to a large literature on redistribution through public provision of goods or in-kind transfers. Nichols and Zeckhauser (1982) show that participation costs can be used to screen for higher-need individuals, while Blackorby and Donaldson (1988) demonstrate that in-kind transfers can be more effective screening devices than cash. Weitzman (1977) shows that random allocation may outperform market allocation when the welfare criterion differs from utilitarianism, and Che et al. (2013) show that even under utilitarian objectives, random allocations with resale can be superior to competitive markets when agents face budget constraints. More recently, Condorelli (2013), Dworczak et al. (2021), and Akbarpour et al. (2024a,b) apply mechanism design to study redistribution under general welfare criteria. However, all these papers consider the public option or transfer program in isolation. In contrast, this paper analyzes a setting in which the public option and a monopolist coexist within a broader mixed market, with no single provider fully determining market outcomes.

In this regard, the most closely related papers are Besley and Coate (1991); Coate et al. (1994); Kang (2023); Kang and Watt (2024). Besley and Coate (1991) show that agents self-select into in-kind transfer programs when a private market is available, and Kang and Watt (2024) studies the optimal design of such transfers in the presence of a competitive private market. However, both papers abstract from the impact of the transfer program on private market prices. Coate et al. (1994), in contrast, examine the competitive effect of in-kind transfers on private market prices, but treat participation in the program as exogenous. Kang (2023) studies how the government's choices regarding the public option's pricing, quality, and allocation affect both the composition of demand in the private market and its price.

However, the central focus across all of these papers is the design of the public option in the presence of a private (and typically, perfectly competitive) market. By contrast, this paper considers the complementary design problem of a monopolist in the presence of a public option that is equally accessible to all agents. Thus, the central focus of this paper is understanding how the private market's profit-maximizing incentives interact with the dual objectives of the public option as a safety net and as a source of competitive pressure.

2. Model

Setup: A unit mass of risk-neutral buyers each have a unit demand for a good. Each buyer's valuation for the good, which is the buyer's private information, is drawn independently from a distribution F over a compact interval $\mathcal{V} \coloneqq [\underline{v}, \bar{v}]$, with $\bar{v} > \underline{v} \geq 0$. The distribution F is assumed to be *regular*: it admits a positive and differentiable density f, and the associated virtual value function $\varphi(v) \coloneqq v - (1 - F(v))/f(v)$ is strictly increasing.[1] A buyer with valuation $v \in \mathcal{V}$ who pays $\mathrm{t} \geq 0$ and receives the good with probability $\mathrm{x} \in [0, 1]$ earns a payoff of $\mathrm{x}v - \mathrm{t}$.

Each buyer may acquire the good from one of two suppliers: a monopolist or a public option. For simplicity, I assume that neither face a cost of production. The monopolist seeks to maximize its expected revenue from selling the good. The public option offers the good for free but it can only supply a mass $k \in (0, 1)$ of goods. When demand for the public option exceeds its capacity, it allocates the good through a random rationing rule. Specifically, if a mass q of buyers demands the public option, each receives the good with probability $\min\{1, k/q\}$.

[1] This is a stronger regularity condition than Myerson (1981), which does not assume differentiability of the density function.

Timing: First, the monopolist selects a selling mechanism. Then, each buyer privately observes her valuation for the good and chooses whether to participate in the monopolist's mechanism or opt for the public option. For those who choose the monopolist, the mechanism determines whether they receive the good and how much they pay. Buyers who fail to acquire the good from the monopolist may then turn to the public option, reflecting its role as a safety net.[2] Finally, the public option allocates the good according to a random rationing rule to buyers who either initially selected it or came to rely on it after failing to purchase from the monopolist.

Mechanisms: Without loss of generality, I restrict attention to direct revelation mechanisms that induce all buyers to initially choose the monopolist. A mechanism is a pair (x, t), where $x : \mathcal{V} \to [0, 1]$ is a measurable function representing the allocation rule and $t : \mathcal{V} \to \mathbb{R}_+$ is a measurable and bounded function representing the transfer rule. A buyer who reports type $\hat{v} \in \mathcal{V}$ pays $t(\hat{v})$ to the monopolist and is allocated a good from the monopolist with probability $x(\hat{v})$. With the complementary probability $1 - x(\hat{v})$, the buyer must instead rely on the public option.

If all buyers participate in the mechanism and report their types truthfully, the mass of buyers who rely on the public option, referred to as the public option's *induced demand*, is given by

$$q(x) := \int_{\mathcal{V}} \big(1 - x(v)\big) dF(v).$$

Accordingly, conditional on relying on the public option, a buyer receives the good with probability $\min\{1, k/q(x)\}$. Thus, a buyer with valuation v who reports $\hat{v}$ (while all other buyers report truthfully) earns an expected payoff of

$$U(\hat{v}, v|x, t) := \left(x(\hat{v}) + \big(1 - x(\hat{v})\big) \cdot \min\left\{1, \frac{k}{q(x)}\right\}\right) v - t(\hat{v}).$$

A mechanism (x, t) is incentive compatible if truthful reporting is optimal for each buyer when all others report truthfully. Formally, (x, t) is incentive compatible if

$$U(v, v|x, t) \geq U(\hat{v}, v|x, t), \ \forall v, \hat{v} \in \mathcal{V}. \tag{IC}$$

[2]In the Online Appendix, I consider the reverse timing: the outcome of the public option is resolved before that of the monopolist's mechanism. In that case, buyers who fail to receive the good from the public option may "top up" their demand by turning to the monopolist.

Furthermore, the mechanism is individually rational if each buyer weakly prefers participating in it rather than relying solely on the public option:

$$U(v, v|x, t) \geq v \cdot \min\left\{1, \frac{k}{q(x)}\right\}, \ \forall v \in \mathcal{V}. \tag{IR}$$

The monopolist's objective is to maximize its expected revenue by offering an incentive compatible and individually rational mechanism.

This setting has three distinctive features: First, while the monopolist can ensure a buyer is allocated the good, it cannot fully exclude a buyer from receiving the good. Second, as reflected in the (IR) constraint, a buyer's outside payoff is type-dependent. Finally, the mechanism determines the level of rationing through the public option's induced demand. Consequently, the mechanism affects buyers' willingness-to-pay for a guaranteed allocation from the monopolist.

When $k = 0$, these three distinctive features disappear and the mechanism design problem reduces to a standard monopoly screening problem. In this case, the revenue-maximizing mechanism is a posted price of $v^M \coloneqq \min\{v \in \mathcal{V} : \varphi(v) \geq 0\}$ (Myerson, 1981).

3. Optimal Mechanism

Let $\mathcal{X}$ denote the space of all measurable allocation rules and let $\mathcal{T}$ denote the space of all measurable and bounded transfer rules. An *optimal mechanism* solves:

$$\max_{(x,t)\in\mathcal{X}\times\mathcal{T}} \int_{\mathcal{V}} t(v)dF(v) \ \text{ s.t. } (x, t) \ \text{ satisfies (IC) and (IR).} \tag{Opt-1}$$

An optimal mechanism (x, t) is *essentially unique* if any $(\hat{x}, \hat{t})$ that solves (Opt-1) satisfies $x(v) = \hat{x}(v)$ and $t(v) = \hat{t}(v)$ for almost all $v \in \mathcal{V}$.

To characterize the solution to (Opt-1), define the function $G : \mathcal{V} \to \mathcal{V}$ by

$$G(v) \coloneqq \varphi(v)F(v) + \int_v^{\bar{v}} \varphi(s)dF(s),$$

which is a strictly increasing function with $\underline{v} = G(\underline{v}) < G(\bar{v}) = \bar{v}$.

Proposition 1 *Let ϑ be the unique value of $v \in \mathcal{V}$ that solves*

$$\frac{k}{F(v)^2} \cdot G(v) = \varphi(v). \tag{1}$$

The essentially unique optimal mechanism is given by the pair (x, t) where

$$x(v) = \begin{cases} 0 & \text{if} \quad v < \vartheta \\ 1 & \text{if} \quad v \geq \vartheta \end{cases} \quad \text{and} \quad t(v) = \begin{cases} 0 & \text{if} \quad v < \vartheta \\ \vartheta \left(1 - \dfrac{k}{F(\vartheta)}\right) & \text{if} \quad v \geq \vartheta \end{cases}.$$

Furthermore, $\vartheta \in (\max\{v^M, F^{-1}(k)\}, \bar{v})$.

Proposition 1 establishes that the optimal mechanism is a posted price that segments consumers into two groups based on their valuation. "High-value" buyers purchase the good from the monopolist, thereby assuring themselves access to the good, even though the public option is available at no monetary cost. Conversely, "low-value" buyers rely only on the public option, accepting the risk of not being rationed a good due to capacity constraints.[3]

The distinction between high- and low-value buyers is determined by a cutoff type $\vartheta \in \mathcal{V}$ that uniquely solves the trade-off captured in (1). To build intuition, consider a monopolist who sells only to consumer types above some cutoff $v \geq \max\{v^M, F^{-1}(k)\}$. The monopolist's expected revenue in this case is given by

$$v(1 - F(v)) - \underbrace{\frac{k}{F(v)} \int_v^{\bar{v}} \varphi(s) dF(s)}_{:= C(v)},$$

where the first term represents the revenue the monopolist would earn in a standard monopoly setting, while the second term captures the revenue loss (compared to the standard setting) due to competition from the public option. Specifically, in a standard screening problem, the monopolist can extract $\varphi(s)$ from a type-s consumer. In the current setting, however, a fraction $k/F(v)$ of this virtual surplus is no longer extractable. Importantly, this fraction is equal to the rationing probability with which

[3] While the mechanism design problem is framed such that all buyers initially approach the monopolist and turn to the public option only as a fallback, the mixed-market outcome under the optimal mechanism can be implemented with the monopolist posting a price and buyers choosing (once and for all) between the monopolist or the public option.

consumers obtain the good from the public option.

The expected revenue above is equivalent to that of a standard monopolist with costly production, where $C(v)$ can be interpreted as the "effective cost" of producing a mass $1-F(v)$ of goods. The function C is differentiable and strictly decreasing with $C(\bar{v})=0$, and its derivative is proportional to G. From this perspective, raising the cutoff v marginally reduces the effective production cost by $C'(v)$, which corresponds to the left-hand side of (1). However, doing so also reduces the monopolist's revenue by $\varphi(v)$ from excluding the cutoff type-v buyer. The optimal cutoff ϑ is thus characterized by equating the marginal cost savings with the marginal loss in revenue.

The optimal cutoff ensures that the public option's induced demand exceeds its capacity, i.e., $F(\vartheta)>k$, leading to rationing. Furthermore, $\vartheta>v^M$, meaning that the public option crowds out some of the monopolist's demand.[4]

4. Comparative Statics

This section examines two sets of comparative statics. First, I analyze how changes in the capacity of a public option affect consumer and producer welfare in a mixed market. Second, I consider how consumer welfare changes when a monopolist enters a market that was previously served only by a public option.

Expanding a Public Option: What is the effect of introducing a public option to a monopoly market, or expanding the capacity of an existing public option in a mixed market? For example, in the United States, where the housing market is primarily supplied by the private sector, how would increasing the public housing supply affect producer and consumer welfare?

To address these questions, I examine how the monopolist and consumer surplus vary with the public option's capacity. For each $k\in(0,1)$, let $\vartheta(k)\in\mathcal{V}$ represent the cutoff type that solves (1). Define

$$\pi(k):=\frac{k}{F\big(\vartheta(k)\big)}$$

[4]In principle, there are incentive compatible and individually rational mechanisms in which the monopolist sells to all types $v\geq v^M$. However, such a mechanism would be suboptimal because a cutoff of v^M does not fully exploit the monopolist's benefit from a congested public option.

as the public option's rationing probability given the monopolist's optimal mechanism, and let

$$p(k) \coloneqq \vartheta(k)\,(1 - \pi(k))$$

represent the revenue-maximizing posted price. The monopolist's surplus is given by

$$\mathcal{P}(k) \coloneqq p(k)\big(1 - F\big(\vartheta(k)\big)\big),$$

and the surplus of a type-v consumer is given by

$$\mathcal{U}(v,k) = \begin{cases} \pi(k) \cdot v & \text{if} \quad v < \vartheta(k) \\ v - p(k) & \text{if} \quad v \geq \vartheta(k) \end{cases} .$$

Finally, let

$$\mathcal{C}(k) \coloneqq \int_{\mathcal{V}} \mathcal{U}(v,k) dF(v)$$

denote the aggregate consumer surplus.

The first result in this section establishes that as the public option's capacity expands, the set of consumers served by the monopolist continuously shrinks.

Proposition 2 *The cutoff type $\vartheta(k)$ is continuous and strictly increasing in k. Furthermore, $\lim_{k\to 0} \vartheta(k) = v^M$ and $\lim_{k\to 1} \vartheta(k) = \bar{v}$.*

Given the continuity of the cutoff type, the associated rationing probability, price, and surplus functions are all continuous. Furthermore, as $k \to 1$, an expanding mass of consumers obtain the good from the public option with diminishing congestion. Hence, when the capacity of the public option is sufficiently large, consumers are strictly better off than in a monopoly-only market (i.e., when $k = 0$).

At the same time, as $k \to 1$, the monopolist serves an increasingly narrow segment of the market and earns vanishing profits. More generally, expanding the capacity of the public option always erodes the monopolist's profits, as formalized in the following proposition.

Proposition 3 *The monopolist's surplus $\mathcal{P}(k)$ is strictly decreasing in k.*

What about consumer welfare? First, consider low-value consumers who rely on the public option. These consumers benefit from a capacity expansion if and only if the public option's rationing probability increases. Here, an expansion generates two opposing effects. First, holding fixed the set of consumers who buy from the monopolist, the additional capacity alleviates congestion at the public option. Second, as established in Proposition 2, the monopolist responds to a capacity expansion by raising the cutoff type, which increases the public option's induced demand.

The following proposition establishes that the first effect always dominates, and thus, low-value consumers unambiguously benefit from an increase in the public option's capacity despite the monopolist's strategic response.[5]

Proposition 4 *For all $k \in (0,1)$ and all $v \leq \vartheta(k)$, $\partial \mathcal{U}(v,k)/\partial k \geq 0$.*

Next, consider high-value consumers who purchase from the monopolist. These consumers benefit from a capacity expansion if and only if it leads the monopoly to lower its price. Again, an expansion generates two opposing effects on the posted price. On the one hand, as implied by Proposition 4, an expansion improves the public option's rationing probability. This makes the public option more attractive for all buyer types, which exerts a downward pressure on the monopolist's price. On the other hand, as established by Proposition 2, the marginal buyer from the monopolist now has a higher valuation and may be willing to pay more for a guaranteed allocation. This creates an opposing upward pressure. The following proposition establishes sharp conditions under which the downward pressure on prices always dominates.

Proposition 5 *Suppose for all $v \in (v^M, \bar{v})$,*

$$\frac{v\, f(v)}{1 - F(v)} + \frac{v\, G'(v)}{G(v)} \geq 2. \tag{2}$$

Then for all $k \in (0,1)$ and all $v > \vartheta(k)$, $\partial \mathcal{U}(v,k)/\partial k \geq 0$. Conversely, if (2) fails for some $v \in (v^M, \bar{v})$, then there exists an open set $K \subseteq (0,1)$ such that for all $k \in K$ and all $v > \vartheta(k)$, $\partial \mathcal{U}(v,k)/\partial k < 0$.

Proposition 5 links the price effect of expanding the public option to two underlying market primitives. The first is the sensitivity of the monopolist's demand to changes in the cutoff type (and thus the posted price), as captured by the first term

[5] The proof of Proposition 4 shows that *almost all* low-value consumers *strictly* benefit from an increase in the public option's capacity.

on the left-hand side of (2). A high elasticity of demand implies that the monopolist cannot sharply raise the cutoff without a substantial loss in its market share. The second is the curvature of the monopolist's effective cost function, reflected in the second term. The higher this term is over $(v^M, \bar{v})$, the more convex the effective cost function, implying that the marginal cost savings from further restricting supply diminish rapidly.

When market demand is sufficiently elastic or the monopolist's effective cost is sufficiently convex, as captured by (2), the monopolist refrains from excessively restricting supply in response to a capacity expansion. In this case, the downward pressure on price arising from the improved outside option dominates the upward pressure from the increased valuation of the marginal buyer, thereby benefiting all high-value consumers.

As Proposition 5 makes clear, whether high-value consumers benefit from a capacity expansion is fully determined by whether the distribution of types above the standard monopoly cutoff satisfies Condition (2). For certain distributions, this condition simplifies substantially:

Corollary 1 *Suppose $v^M = \underline{v}$ and that F has an increasing hazard rate. Then for all $k \in (0,1)$ and all $v > \vartheta(k)$, $\partial \mathcal{U}(v,k)/\partial k \geq 0$ if and only if $f(\underline{v})\,\underline{v} \geq 2$.*

More broadly, Proposition 4 and Proposition 5 taken together imply that a capacity expansion is especially likely to improve aggregate consumer welfare when a sufficiently large fraction of buyers are excluded by a monopoly-only market. In this case, even when Condition (2) fails, the welfare gains accruing to low-value consumers may offset any losses experienced by high-value consumers.

Let us conclude this section with two examples: Consider first the case where F is the uniform distribution over the unit interval. In this case, $v^M = 0.5$, and the left-hand side of (2) becomes

$$\frac{v}{1-v} + 2.$$

Hence, high-value consumers always benefit as k increases in this case. In particular, as shown in Figure 1a, introducing a public option with a small capacity k to an intially monopoly-only market raises the surplus of each buyer type. More generally, Figure 1b shows that the aggregate consumer surplus $\mathcal{C}(k)$ is strictly increasing.

Next, consider the case where F is the uniform distribution over the interval $[1, 2]$. In this case, $v^M = \underline{v} = 1$, and applying Corollary 1, we can conclude that high-value

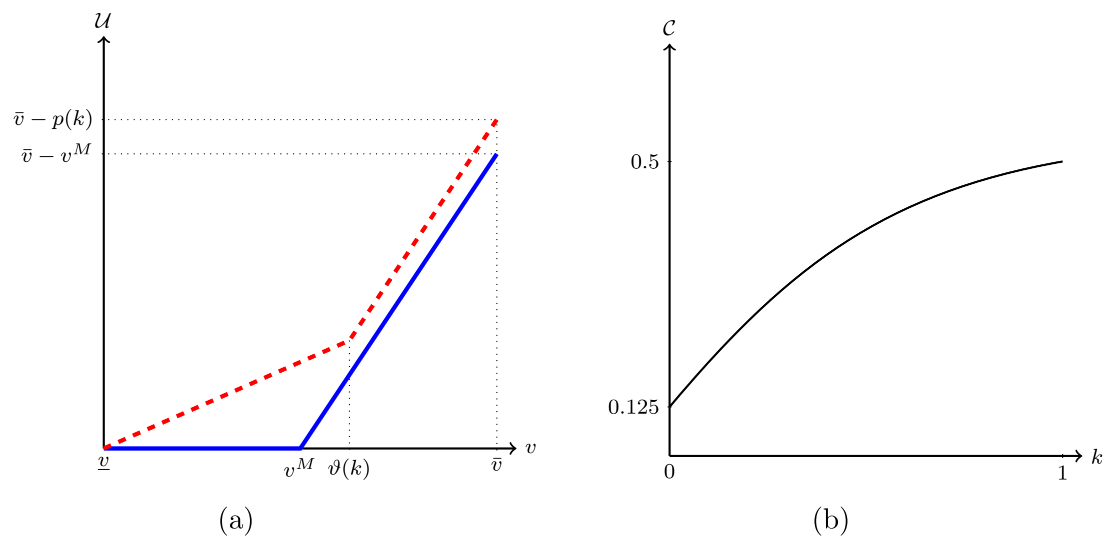

Figure 1: In both panels, consumer values are uniformly distributed over the unit interval, so $\underline{v} = 0$, $\bar{v} = 1$, and $v^M = 0.5$. Panel (a) compares each type-v buyer's payoff in a monopoly-only market—$\mathcal{U}(v, 0)$, shown in the solid blue line—to that of a mixed market—$\mathcal{U}(v, k)$ for a small $k \in (0, 1)$, shown in the dashed red line. Panel (b) depicts how the aggregate consumer surplus varies with the capacity.

buyers do not always benefit from a capacity expansion. Specifically, the left-hand side of (2) becomes

$$\frac{v}{2 - v} + \frac{2v(v - 1)}{v^2 - 2v + 2},$$

which is strictly less than 2 over the interval $(\underline{v}, \hat{v})$, where $\hat{v} \approx 1.206$. In this case, as shown in Figure 2a, introducing a public option with a small capacity k to an initially monopoly-only market lowers the surplus of most buyer types. As a result, the aggregate consumer surplus $\mathcal{C}(k)$ is non-monotone, as shown in Figure 2b.

Introducing a Monopolist What is the effect of introducing a profit-maximizing monopolist into a market served exclusively by a congested public option? This question is particularly relevant for markets like the UK's National Health Service, where public provision is widespread but allocation is rationed due to capacity constraints.

The following proposition establishes that introducing a monopoly to such a market improves the welfare of each consumer type.[6]

[6] In fact, the proof of Proposition 6 shows that almost all consumers *strictly* benefit from introducing a monopolist to the public-option-only market.

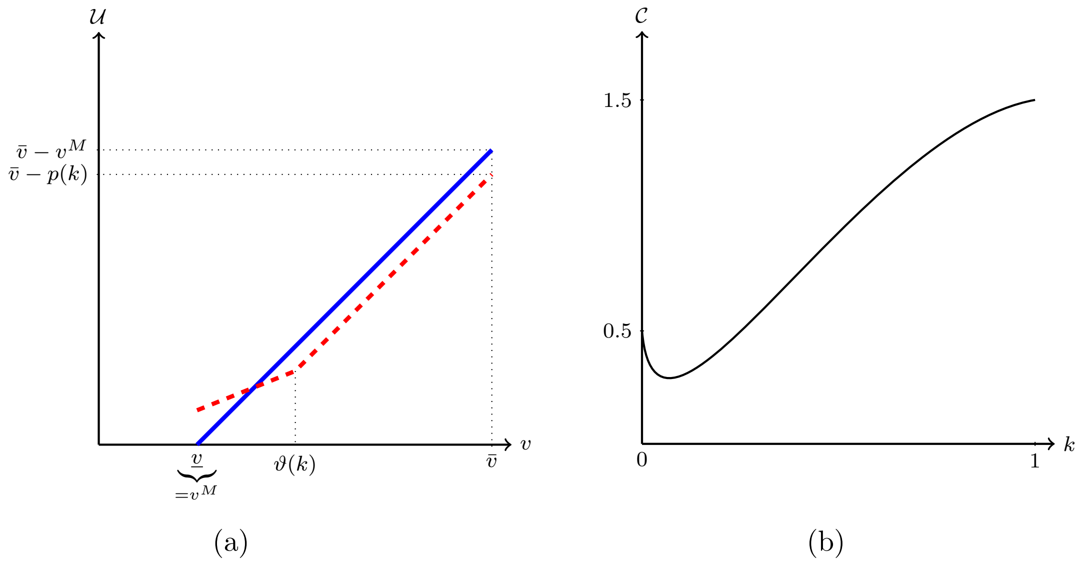

Figure 2: In both panels, consumer values are uniformly distributed over $[1, 2]$, so $\underline{v} = v^M = 1$ and $\bar{v} = 2$. Panel (a) compares each type-v buyer's payoff in a monopoly-only market—$\mathcal{U}(v, 0)$, shown in the solid blue line—to that of a mixed market—$\mathcal{U}(v, k)$ for a small $k \in (0, 1)$, shown in the dashed red line. Panel (b) depicts how the aggregate consumer surplus varies with the capacity.

Proposition 6 *For all $k \in (0, 1)$ and all $v \in \mathcal{V}$, $\mathcal{U}(v, k) \geq k \cdot v$.*

The intuition behind Proposition 6 is straightforward. High-value consumers who purchase the good from the monopolist reveal a preference for guaranteed allocation at the posted price rather than a free allocation from a rationed public option. As a result, they are better off than under the public option alone. Moreover, demand at the public option decreases because high-value types no longer rely on it. This improves the rationing probability and thus raises the surplus for low-value consumers who continue to rely on the public option. Therefore, all consumer types benefit from the introduction of the monopolist.

5. Conclusion

This paper studies how a profit-maximizing monopolist responds to competition from a capacity-constrained public option. The monopolist strategically restricts supply to exacerbate rationing at the public option, thereby increasing consumers' willingness to pay for guaranteed access. In equilibrium, high-valuation consumers purchase from the monopolist, while low-valuation consumers rely on the rationed public option.

The analysis yields several policy implications. First, interventions that introduce a public option into a monopolistic market or expand the capacity of an existing public option within a mixed market can reduce consumer welfare unless the intervention is sufficiently large. While any such intervention always benefits low-valuation consumers who rely on the public option, a small intervention may induce the monopolist to raise its price, thereby reducing surplus for high-valuation consumers. Second, I show that introducing a profit-maximizing monopolist to a market served only by a capacity-constrained public option unambiguously improves consumer surplus for all types.

These findings highlight the interplay between public and private providers. Importantly, policy targeting the public provision of goods and services in mixed markets should account for the strategic responses of private firms. The framework developed here also provides a tractable foundation for future work, for example, studying the design of optimal tax schemes to finance the costly provision of public goods in mixed markets.

Appendix

Proof of Proposition 1. Given an allocation rule $x \in \mathcal{X}$, define the *transformed allocation probability* $\chi(\cdot|x) : \mathcal{V} \to [0,1]$ by

$$\chi(\hat{v}|x) \coloneqq x(\hat{v}) \left(1 - \min\left\{1, \frac{k}{q(x)}\right\}\right).$$

We can then express the payoff, net of the outside option, of a type-v buyer from reporting $\hat{v}$ as

$$U(\hat{v}, v|x, t) - v \cdot \min\left\{1, \frac{k}{q(x)}\right\} = \chi(\hat{v}|x)v - t(\hat{v}).$$

A mechanism (x, t) satisfies (IC) if and only if the pair $(\chi(\cdot|x), t)$ satisfies

$$\chi(v|x)v - t(v) \geq \chi(\hat{v}|x)v - t(\hat{v}), \ \ \forall v, \hat{v} \in \mathcal{V}, \tag{IC$'$}$$

and similarly, the mechanism satisfies (IR) if and only if

$$\chi(v|x)v - t(v) \geq 0, \ \ \forall v \in \mathcal{V}. \tag{IR$'$}$$

Written in this format, it is evident that we can leverage Myerson (1981) to conclude that a mechanism (x, t) satisfies (IC) if and only if $\chi(\cdot|x)$ is non-decreasing, and for all $v \in \mathcal{V}$,

$$t(v) = \chi(v|x)v - \int_{\underline{v}}^{v} \chi(s|x)ds - u(\underline{v}|x, t), \tag{3}$$

where $u(\underline{v}|x, t) \coloneqq \chi(\underline{v}|x)\underline{v} - t(\underline{v})$. Furthermore, an incentive-compatible (x, t) satisfies (IR) if and only if $u(\underline{v}|x, t) \geq 0$.

Consequently, the monopolist's revenue from an incentive-compatible mechanism (x, t) can be expressed as

$$R(x, t) \coloneqq \int_{\mathcal{V}} \chi(v|x)\varphi(v)dF(v) - u(\underline{v}|x, t).$$

Clearly, if (x, t) is an optimal mechanism, then $u(\underline{v}|x, t) = 0$.

Moreover, if (x, t) is an optimal mechanism, then $q(x) > k$. To see why, suppose, for the sake of contradiction, that $q(x) \leq k$. By definition, $\chi(v|x) = 0$ for all $v \in \mathcal{V}$, which further implies that $t(v) = 0$ for all $v \in \mathcal{V}$. Consequently, the monopolist's expected revenue is $R(x, t) = 0$. However, consider an alternative mechanism $(\hat{x}, \hat{t}) \in \mathcal{X} \times \mathcal{T}$, where $\hat{x}(v) = \mathbb{1}[v \geq \bar{v} - \epsilon]$ for $\epsilon > 0$ and $\hat{t}$ is given by (3) with $u(\underline{v}|\hat{x}, \hat{t}) = 0$. The monotonicity of $\hat{x}$ implies that $\chi(\cdot|\hat{x})$ is non-decreasing. Hence, $(\hat{x}, \hat{t})$ is incentive-compatible and individually rational. Furthermore, for ϵ small enough, $q(\hat{x}) > k$ and $\bar{v} - \epsilon > v^M$. It is then straightforward to see that $R(\hat{x}, \hat{t}) > 0$. Thus, to avoid the contradiction, an optimal mechanism (x, t) must satisfy $q(x) > k$.

We can now decompose the optimization problem (Opt-1) into an equivalent nested problem in which the monopolist first maximizes its profits for a given level of induced demand $Q > k$ at the public option, and then optimizes over the induced demand. Formally, the monopolist solves:

$$\max_{Q \in (k,1]} \left\{ \max_{x \in \mathcal{X}} \left(1 - \frac{k}{Q}\right) \int_{\mathcal{V}} x(v)\varphi(v)dF(v) \ \text{ s.t. } \ x \text{ is non-decreasing, and } q(x) = Q \right\}.$$

Consider first the inner problem for some fixed $Q \in (k, 1]$. Define $v^Q \coloneqq F^{-1}(Q)$ as the Q^{th}-quantile type. Since the inner problem is a constrained linear programming problem, it is clear that $x(v) = \mathbb{1}[v \geq v^Q]$ is the essentially unique maximizer.[7] Thus, the value function of the inner problem is $(1 - k/Q) \int_{v^Q}^{\bar{v}} \varphi(v)dF(v)$.

Next, consider the outer optimization problem. Since there is a one-to-one mapping between the induced demand Q and the quantile type v^Q, the outer problem is equivalent to:

$$\max_{v \in (F^{-1}(k), \bar{v}]} \mathcal{R}(v) \coloneqq \left(1 - \frac{k}{F(v)}\right) \int_{v}^{\bar{v}} \varphi(s)dF(s). \tag{Opt-2}$$

Lemma 1 (Opt-2) *has a unique maximizer* $\vartheta \in (F^{-1}(k), \bar{v})$*, and it is characterized by the unique solution to* (1)*. Furthermore,* $\vartheta > v^M$*.*

Proof. Since F is regular, $\mathcal{R}(\cdot)$ is continuous and differentiable over the interval $(F^{-1}(k), \bar{v})$ with

$$\mathcal{R}'(v) = f(v)\left[\frac{k}{F(v)^2}G(v) - \varphi(v)\right] \equiv \frac{f(v)}{F(v)} \underbrace{\left[\frac{k}{F(v)}G(v) - \varphi(v)F(v)\right]}_{:=r(v)}.$$

[7] Section 1 of the Online Appendix establishes this fact in a more general setting.

Notice that $\lim_{v \to F^{-1}(k)} \mathcal{R}(v) = \mathcal{R}(\bar{v}) = 0$. Thus, by Rolle's Theorem, there exists a cutoff type $\vartheta \in (F^{-1}(k), \bar{v})$ that solves (1).

Let us next show that any cutoff type $\vartheta \in (F^{-1}(k), \bar{v})$ that solves (1) satisfies $\vartheta > v^M$. This is trivially true if $v^M \leq F^{-1}(k)$, so let us focus instead on the case that $v^M > F^{-1}(k)$. Since $F^{-1}(k) > \underline{v}$ for $k > 0$, this implies that $v^M > \underline{v}$. This in turn implies $\varphi(v^M) = 0$. From (1), $\varphi(\vartheta) = (k/F(\vartheta)^2)G(\vartheta) > 0$. Hence, by strict monotonicity of the virtual surplus function, $\vartheta > v^M$.

Finally, let us show that ϑ is the unique solution to (1), or equivalently, the unique value with $r(\vartheta) = 0$. For all $v \leq F^{-1}(k)$,

$$r(v) \geq G(v) - \varphi(v)F(v) = \int_v^{\bar{v}} \varphi(s)dF(s) = v\big(1 - F(v)\big) > 0,$$

and

$$r(\bar{v}) = -\varphi(\bar{v})(1-k) < 0.$$

Hence, all solutions to (1) must be in the interval $(\max\{v^M, F^{-1}(k)\}, \bar{v})$. Moreover, over this interval, the mapping $v \mapsto r(v)$ is continuous and strictly decreasing so that $\mathcal{R}$ is strictly concave. As a result, ϑ is the unique solution to (1) and the unique maximizer to (Opt-2). ■

The characterization of the essentially unique optimal mechanism in Proposition 1 then follows immediately from Lemma 1. ■

Proof of Proposition 2. For a given type $v \in \mathcal{V}$ and capacity $k \in (0,1)$, define

$$r(v,k) := \frac{k}{F(v)} \cdot G(v) - \varphi(v)F(v),$$

which is continuous in both arguments. From the proof of Lemma 1, for each $k \in (0,1)$, there exists a unique type $\vartheta(k) \in (\max\{v^M, F^{-1}(k)\}, \bar{v})$ such that $r(\vartheta(k), k) = 0$. This immediately establishes the continuity of the mapping $k \mapsto \vartheta(k)$ over $(0,1)$.

For each k, the mapping $v \mapsto r(v,k)$ is strictly decreasing over $(\max\{v^M, F^{-1}(k)\}, \bar{v})$. Furthermore, for each $v \in (\max\{v^M, F^{-1}(k)\}, \bar{v})$, $\partial r(v,k)/\partial k = G(v)/F(v) > 0$, where the inequality follows because G is a strictly increasing function with $G(v) > G(\underline{v}) = \underline{v} \geq 0$. Thus, for any $k', k'' \in (0,1)$ with $k'' > k'$,

$$0 = r(\vartheta(k'), k') = r(\vartheta(k''), k'') > r(\vartheta(k''), k'),$$

which implies that $\vartheta(k'') > \vartheta(k')$, as desired.

To establish the limiting results, let $\vartheta(1) := \lim_{k\to 1} \vartheta(k)$ and $\vartheta(0) := \lim_{k\to 0} \vartheta(k)$. For all $k \in (0,1)$, $\vartheta(k) \in (F^{-1}(k), \bar{v})$. Thus, $\bar{v} = \lim_{k\to 1} F^{-1}(k) \le \vartheta(1) \le \bar{v}$, establishing the first limit result. Additionally, $\vartheta(k) > v^M$ for all $k \in (0,1)$, which implies $\vartheta(0) \ge v^M$. Additionally, note that for all $k \in (0,1)$,

$$0 = r(\vartheta(k), k) < \frac{k}{F(v^M)} \cdot G(\vartheta(k)) - \varphi(\vartheta(k))F(\vartheta(k)).$$

Taking the limit as $k \to 0$ implies that $0 \le -\varphi(\vartheta(0))F(\vartheta(0))$. Thus, we must have $\varphi(\vartheta(0)) \le 0$, which by the definition of v^M, implies that $\vartheta(0) \le v^M$. We can therefore conclude that $v^M = \vartheta(0)$, as desired. ■

Proof of Proposition 3. For a given cutoff $v \in \mathcal{V}$ and capacity $k \in (0,1)$, define the monopolist's revenue by

$$\mathcal{R}(v,k) := \left(1 - \frac{k}{F(v)}\right) \int_v^{\bar{v}} \varphi(s) dF(s),$$

which is a continuous and differentiable function. Additionally, for each $k \in (0,1)$, $\mathcal{P}(k) = \mathcal{R}(\vartheta(k), k) = \max_{v\in[F^{-1}(k),\bar{v}]} \mathcal{R}(v,k)$.

Given that the mapping $k \mapsto \vartheta(k)$ is continuous and strictly increasing (Proposition 2), it is differentiable almost everywhere. Hence, the value function $k \mapsto \mathcal{P}(k)$ is also differentiable almost everywhere. By the envelope theorem,

$$\mathcal{P}'(k) = \frac{-1}{F(\vartheta(k))} \int_{\vartheta(k)}^{\bar{v}} \varphi(v) dF(v) < 0,$$

for almost all $k \in (0,1)$. Hence, the mapping $k \mapsto \mathcal{P}(k)$ is strictly decreasing, concluding the proof. ■

Proof of Proposition 4. Since the mapping $k \mapsto \vartheta(k)$ is continuous and strictly increasing (Proposition 2), it is almost everywhere differentiable. In fact, since f is assumed to be differentiable (recall the stronger notion of regularity), the mapping $k \mapsto \vartheta(k)$ is *everywhere* differentiable, which in turn implies that $k \mapsto \pi(k)$ is also differentiable.

Given any $k \in (0,1)$, $\partial\mathcal{U}(v,k)/\partial k = \pi'(k) \cdot v$ for $v < \vartheta(k)$, and $\partial\mathcal{U}(\vartheta(k),k)/\partial k = \pi'(k) \cdot \vartheta(k) + \pi(k)\vartheta'(k)$. Hence, it suffices to prove that $\pi'(k) \ge 0$ for all $k \in (0,1)$ to establish the desired result.

To that end, recall the function $r(v,k)$ from the proof of Proposition 2. For each $k \in (0,1)$, the cutoff type $\vartheta(k)$ is implicitly defined by $r\big(\vartheta(k),k\big)=0$, or equivalently, $\pi(k)G\big(\vartheta(k)\big)=\varphi\big(\vartheta(k)\big)F\big(\vartheta(k)\big)$. By implicitly differentiating,

$$\pi'(k)=\frac{\vartheta'(k)}{G\big(\vartheta(k)\big)}\Bigg(\varphi\big(\vartheta(k)\big)f\big(\vartheta(k)\big)+G'\big(\vartheta(k)\big)\big(1-\pi(k)\big)\Bigg)>0,$$

where the inequality follows because $k\mapsto\vartheta(k)$ and $v\mapsto G(v)$ are strictly increasing functions, and because $\varphi\big(\vartheta(k)\big)>\varphi(v^M)\geq 0$ and $G\big(\vartheta(k)\big)>G(\underline{v})=\underline{v}\geq 0$. Hence, the mapping $k\mapsto\pi(k)$ is strictly increasing, which concludes the proof. ■

Proof of Proposition 5. Since $k\mapsto\vartheta(k)$ is differentiable under the assumption that f is differentiable, the mapping $k\mapsto p(k)$ is also differentiable.

If $p'(k)\leq 0$ for all $k\in(0,1)$, then for all $k\in(0,1)$ and all $v>\vartheta(k)$, $\partial\mathcal{U}(v,k)/\partial k\geq 0$. Conversely, if $p'(k)>0$ for some $k'\in(0,1)$, then by continuity, there exists a neighborhood $K\coloneqq(k'-\delta,k'+\delta)$ for $\delta>0$ small enough such that $p(\cdot)$ is increasing over K. In this case, for all $k\in K$ and all $v>\vartheta(k)$, $\partial\mathcal{U}(v,k)/\partial k<0$.

Notice that $p'(k)\leq 0$ is equivalent to

$$\begin{aligned}
&\pi'(k)\geq\frac{\vartheta'(k)}{\vartheta(k)}\big(1-\pi(k)\big)\\
\Leftrightarrow&\frac{\vartheta'(k)}{G\big(\vartheta(k)\big)}\Bigg(\varphi\big(\vartheta(k)\big)f\big(\vartheta(k)\big)+G'\big(\vartheta(k)\big)\big(1-\pi(k)\big)\Bigg)\geq\frac{\vartheta'(k)}{\vartheta(k)}\big(1-\pi(k)\big)\\
\Leftrightarrow&\varphi\big(\vartheta(k)\big)\geq\frac{1-F\big(\vartheta(k)\big)}{f\big(\vartheta(k)\big)}\left(1-\vartheta(k)\cdot\frac{G'\big(\vartheta(k)\big)}{G\big(\vartheta(k)\big)}\right)\\
\Leftrightarrow&\vartheta(k)\cdot\frac{f\big(\vartheta(k)\big)}{1-F\big(\vartheta(k)\big)}+\vartheta(k)\cdot\frac{G'\big(\vartheta(k)\big)}{G\big(\vartheta(k)\big)}\geq 2, \qquad (4)
\end{aligned}$$

where the first line follows from the definition of $p(k)$, the second follows from the characterization of $\pi'(k)$ is the proof of Proposition 4, the third line follows because $\pi(k)G\big(\vartheta(k)\big)=\varphi\big(\vartheta(k)\big)F\big(\vartheta(k)\big)$ by (1), which can be equivalently written as $(1-\pi(k))G\big(\vartheta(k)\big)=\vartheta(k)\big(1-F(\vartheta(k))\big)$, and the last line follows from algebraic manipulation.

Therefore, $p'(k)\leq 0$ if and only if (4) holds. From Proposition 2, we know that

$k \mapsto \vartheta(k)$ is continuous and its image is $(v^M, \bar{v})$. Thus, (4) holds for all $k \in (0,1)$ if and only if (2) holds for all $v \in (v^M, \bar{v})$. ■

Proof of Corollary 1. ("If" direction): Suppose $f(\underline{v})\,\underline{v} \geq 2$. Then for any $v \in \mathcal{V}$, we have

$$2 \leq \frac{v\,f(v)}{1 - F(v)} \leq \frac{v\,f(v)}{1 - F(v)} + \frac{v\,G'(v)}{G(v)},$$

where the first inequality follows from the fact that F has an increasing hazard rate, and the second inequality follows from the fact that G is an increasing and non-negative function. Hence, Condition (2) is satisfies for all $v \in \mathcal{V}$, and the desired result follows from Proposition 5.

("Only-if" direction): Suppose $f(\underline{v})\,\underline{v} < 2$. Then

$$2 > \frac{v^M\,f(v^M)}{1 - F(v^M)} + \frac{v^M\,G'(v^M)}{G(v^M)},$$

where the inequality follows from the fact that $v^M = \underline{v}$ and $G'(\underline{v}) = \varphi'(\underline{v})F(\underline{v}) = 0$. By continuity, there exists an interval $(v^M, v^M + \delta)$ for $\delta > 0$ small enough over which Condition (2) fails, and the desired result follows from Proposition 5. ■

Proof of Proposition 6. For each $k \in (0,1)$ and $v \in \mathcal{V}$,

$$\mathcal{U}(v,k) \geq \min\left\{\frac{k}{F(\vartheta(k))}, 1\right\} \cdot v \geq k \cdot v,$$

where the first inequality follows from (IR), and the last follows because $\vartheta(k) > F^{-1}(k)$ and $v \geq \underline{v} \geq 0$. Furthermore, since $\vartheta(k) < \bar{v}$, the last inequality is strict for all $v > 0$. ■

Online Appendix

A General Model of Public Option This section generalizes the baseline model in two directions. First, I now allow for public option's good to be of lower quality than the one supplied by the monopolist. Second, the public option now supplies the good for a subsidized price. Formally, the monopolist supplies a good of quality $\theta_m > 0$, and seeks to maximize its profit. The public option offers a good of quality $\theta_p \in (0, \theta_m]$ at an exogenously fixed price of $\rho \geq 0$. Without loss of generality, normalize $\theta_m = 1$ and denote $\theta_p/\theta_m \coloneqq \theta \in (0, 1]$. A buyer with valuation $v \in \mathcal{V}$ who pays $\mathrm{t} \geq 0$ and receives a good of quality $\tilde{\theta} \in \{\theta, 1\}$ with probability $\mathrm{x} \in [0, 1]$ earns a payoff of $\mathrm{x}\tilde{\theta}v - \mathrm{t}$.

If $\rho > \theta \underline{v}$, then buyer types $v \in [\underline{v}, \min\{v^M, \rho/\theta\})$ would be excluded by both the monopolist and the public option.[8] In this case, the market outcome would be equivalent to one in which buyer valuations are drawn from a truncated distribution supported on $[\min\{v^M, \rho/\theta\}, \bar{v}]$. For expositional simplicity, I therefore assume $\rho \leq \theta \underline{v}$ so that no buyer types are excluded by the public option.

As in the baseline model, a mechanism is a pair $(x, t) \in \mathcal{X} \times \mathcal{T}$, where a buyer who reports type $\hat{v} \in \mathcal{V}$ pays $t(\hat{v})$ to the monopolist and is allocated a good from the monopolist with probability $x(\hat{v})$. With the complementary probability $1 - x(\hat{v})$, the buyer must instead rely on the public option.

If all buyers report their types truthfully, the public option's induced demand is

$$q(x) \coloneqq \int_{\mathcal{V}} \big(1 - x(v)\big) dF(v).$$

Accordingly, a buyer with valuation v who reports $\hat{v}$ while all other buyers report truthfully earns an expected payoff of

$$U(\hat{v}, v | x, t) \coloneqq x(\hat{v})v + \big(1 - x(\hat{v})\big) \cdot \min\left\{1, \frac{k}{q(x)}\right\} (\theta v - \rho) - t(\hat{v}).$$

A mechanism (x, t) is incentive compatible if

$$U(v, v | x, t) \geq U(\hat{v}, v | x, t), \ \forall v, \hat{v} \in \mathcal{V}, \tag{IC}$$

[8] Recall that v^M denotes the monopoly price in the standard monopoly setting.

and the mechanism is individually rational if

$$U(v, v|x, t) \geq \min\left\{1, \frac{k}{q(x)}\right\}(\theta v - \rho), \ \forall v \in \mathcal{V}. \tag{IR}$$

The monopolist's objective is to maximize its expected revenue by offering an incentive compatible and individually rational mechanism. The monopolist therefore solves the following mechanism design problem:

$$\max_{(x,t)\in\mathcal{X}\times\mathcal{T}} \int_{\mathcal{V}} t(v)dF(v) \ \text{ s.t. } (x, t) \ \text{ satisfies (IC) and (IR).} \tag{MD}$$

Given a mechanism (x, t), define the *transformed allocation rule* by

$$\chi(\hat{v}|x) := x(\hat{v})\left(1 - \theta \cdot \min\left\{\frac{k}{q(x)}, 1\right\}\right),$$

and define the *transformed transfer rule* by

$$\tau(\hat{v}|x, t) := t(\hat{v}) - \rho x(\hat{v}) \cdot \min\left\{\frac{k}{q(x)}, 1\right\}.$$

Notice that payoffs net of the outside option can now be written as

$$U(\hat{v}, v|x, t) - (\theta v - \rho) \cdot \min\left\{\frac{k}{q(x)}, 1\right\} = \chi(\hat{v}|x)v - \tau(\hat{v}|x, t).$$

Thus, a mechanism (x, t) satisfies (IC) if and only if

$$\chi(v|x)v - \tau(v|x, t) \geq \chi(\hat{v}|x)v - \tau(\hat{v}|x, t), \ \ \forall v, \hat{v} \in \mathcal{V}. \tag{IC$'$}$$

Similarly, (x, t) satisfies (IR) if and only if

$$\chi(v|x)v - \tau(v|x, t) \geq 0, \ \ \forall v \in \mathcal{V}. \tag{IR$'$}$$

Thus, leveraging Myerson (1981), a mechanism (x, t) satisfies (IC) if and only if

(*a*) $\chi(\cdot|x)$ is non-decreasing, and

(b) For all $v \in \mathcal{V}$,

$$t(v) = \chi(v|x)v + \rho x(v) \cdot \min\left\{\frac{k}{q(x)}, 1\right\} - \int_{\underline{v}}^{v} \chi(s|x)ds - u(\underline{v}|x,t).$$

Moreover, an incentive-compatible mechanism (x,t) satisfies (IR) if $\underline{u}(x,t) \coloneqq \chi(\underline{v}|x)\underline{v} - \tau(\underline{v}|x,t) \geq 0$.

Consequently, the monopolist's revenue from an incentive-compatible mechanism (x,t) can be expressed as

$$\begin{aligned}
&\int_{\mathcal{V}} t(v)dF(v) \\
&= \int_{\mathcal{V}} \chi(v|x)\varphi(v)dF(v) + \rho \cdot \min\left\{\frac{k}{q(x)}, 1\right\} \int_{\mathcal{V}} x(v)dF(v) - \underline{u}(x,t) \\
&= \int_{\mathcal{V}} x(v)\left[\varphi(v)\left(1 - \theta \cdot \min\left\{\frac{k}{q(x)}, 1\right\}\right) + \rho \cdot \min\left\{\frac{k}{q(x)}, 1\right\}\right] dF(v) - \underline{u}(x,t).
\end{aligned}$$

Clearly, if (x,t) is an optimal mechanism, then $\underline{u}(x,t) = 0$.

Given an allocation rule $x \in \mathcal{X}$ and an arbitrary induced demand $Q \in [0,1]$, define

$$R(x,Q) \coloneqq \int_{\mathcal{V}} x(v)\left[\varphi(v)\left(1 - \theta \cdot \min\left\{\frac{k}{Q}, 1\right\}\right) + \rho \cdot \min\left\{\frac{k}{Q}, 1\right\}\right] dF(v)$$

We can decompose the optimization problem (MD) into the following equivalent nested problem:

$$\max_{Q \in (k,1]} \left\{\max_{x \in \mathcal{X}} R(x,Q) \quad \text{s.t.} \quad \chi(\cdot|x) \text{ is non-decreasing, and } q(x) = Q\right\}.$$

Let us first solve the inner constrained linear programming problem. To that end, fix some $Q \in [0,1]$. First, notice that if

$$1 - \theta \cdot \min\left\{\frac{k}{Q}, 1\right\} = 0,$$

then the problem is trivial. The optimal solution is any allocation rule $x \in \mathcal{X}$ (monotone or not) such that $q(x) = Q$.

Next, consider instead the case where

$$1 - \theta \cdot \min\left\{\frac{k}{Q}, 1\right\} > 0.$$

In this case, $\chi(\cdot|x)$ is non-decreasing if and only if x is non-decreasing. Let $\mathcal{X}_M$ be the set of non-decreasing allocation functions, which is a convex and compact subset of the set of integrable allocation functions. Define the subset

$$\mathcal{X}_M^Q := \{x \in \mathcal{X}_M : q(x) = Q\}.$$

Recall that $v^Q := F^{-1}(Q)$ is defined as the Q^{th}-quantile type. Define the allocation rule $x^Q(v) := \mathbb{1}[v \geq v^Q]$, and notice that $x^Q \in \mathcal{X}_M^Q$, so $\mathcal{X}_M^Q$ is non-empty. Moreover, since the mapping $x \mapsto q(x)$ is linear and continuous, the subset $\mathcal{X}_M^Q$ is also a convex and compact set. Therefore, the inner linear programming problem given by

$$\max_{x \in \mathcal{X}_M^Q} R(x, Q) \tag{IP}$$

attains its maximum at an extreme point of $\mathcal{X}_M^Q$. From Winkler (1988) (Proposition 2.1), x is an extreme point of $\mathcal{X}_M^Q$ if there exists a weight $\alpha \in [0,1]$ and step functions $x_1(v) = \mathbb{1}[v \geq v_1]$ and $x_2(v) = \mathbb{1}[v \geq v_2]$ with cutoffs $v_1, v_2 \in \mathcal{V}$ such that $x = \alpha x_1 + (1-\alpha)x_2$ and $q(x) = Q$.

Suppose the inner problem attains its maximum at $x^* = \alpha x_1 + (1-\alpha)x_2$ where $\alpha \in (0,1)$ and the step functions x_1, x_2 have cutoffs $v_1 < v_2$, respectively. Since $q(\cdot)$ is linear, we have

$$q(x^*) = \alpha q(x_1) + (1-\alpha)q(x_2) = \alpha F(v_1) + (1-\alpha)F(v_2).$$

At the same time, the fact that $x^* \in \mathcal{X}_M^Q$ implies that $q(x^*) = q(x^Q) = F(v^Q)$. Equating the two expressions for $q(x^*)$, we have that $v^Q \in (v_1, v_2)$ and

$$\alpha = \frac{F(v_2) - F(v^Q)}{F(v_2) - F(v_1)}.$$

The monopolist's profit from implementing x^* is then given by

$$\begin{aligned} R(x^*,Q) = &\left(1-\theta\cdot\min\left\{\frac{k}{Q},1\right\}\right)\left[\int_{v_2}^{\bar{v}}\varphi(v)dF(v)+\alpha\int_{v_1}^{v_2}\varphi(v)dF(v)\right] \\ &+\rho\cdot\min\left\{\frac{k}{Q},1\right\}\left[1-\underbrace{\Big(\alpha F(v_1)+(1-\alpha)F(v_2)\Big)}_{=F(v^Q)}\right] \\ <&\left(1-\theta\cdot\min\left\{\frac{k}{Q},1\right\}\right)\int_{v^Q}^{\bar{v}}\varphi(v)dF(v)+\rho(1-Q)\cdot\min\left\{\frac{k}{Q},1\right\} \\ =&R(x^Q,Q), \end{aligned}$$

where the inequality follows because

$$1-\theta\cdot\min\left\{\frac{k}{Q},1\right\}>0$$

by assumption, and because the strict monotonicity of φ implies

$$\mathbb{E}_F\big[\varphi\big|v\in[v_1,v_2]\big]<\mathbb{E}_F\big[\varphi\big|v\in[v^Q,v_2]\big].$$

Consequently, x^Q is the essentially unique allocation rule in $\mathcal{X}_M^Q$ at which the inner problem attains its maximum.

Let us next solve the outer optimization problem. Since $Q=F(v^Q)$ by construction, choosing $Q\in[0,1]$ is equivalent to choosing a cutoff type $v\in\mathcal{V}$. Hence, the outer problem is equivalent to:

$$\max_{v\in\mathcal{V}}\left(1-\theta\cdot\min\left\{\frac{k}{F(v)},1\right\}\right)\int_{v}^{\bar{v}}\varphi(s)dF(s)+\rho(1-F(v))\cdot\min\left\{\frac{k}{F(v)},1\right\}$$

$$\equiv\max_{v\in\mathcal{V}}(1-F(v))\left[v\left(1-\theta\cdot\min\left\{\frac{k}{F(v)},1\right\}\right)+\rho\cdot\min\left\{\frac{k}{F(v)},1\right\}\right] \tag{OP}$$

Generically, (OP) has a unique solution $v^*\in\mathcal{V}$. The optimal mechanism is then given by

$$x(v) = \begin{cases} 0 & \text{if} \quad v < v^* \\ 1 & \text{if} \quad v \geq v^* \end{cases}$$

and

$$t(v) = \begin{cases} 0 & \text{if} \quad v < v^* \\ v^* \left(1 - \theta \cdot \min\left\{\frac{k}{F(v^*)}, 1\right\}\right) + \rho \cdot \min\left\{\frac{k}{F(v^*)}, 1\right\} & \text{if} \quad v \geq v^* \end{cases}.$$

Thus, a posted price mechanism remains optimal in this general setting. Moreover, when θ is close to 1 and ρ is close to 0, the optimal cutoff v^* that solves (OP) is close to the optimal cutoff ϑ that solves (Opt-2). Hence, the posted price of Proposition 1 along with the associated comparative statics results in the baseline model extend to the general setting presented here.

However, the general setting also allows for other outcomes depending on the parameters of the model. For example, if both $k, \theta \approx 1$ and ρ is bounded away from zero (which, given the assumption that $\rho \leq \theta \underline{v}$, requires that $\underline{v} \gg 0$), the optimal cutoff is given by

$$v^* = \min\{v \in \mathcal{V} : (1 - \theta)\varphi(v) + \rho \geq 0\}.$$

In this case, $v^* \leq v^M$, implying that the presence of the public option leads to a crowding-in effect. Intuitively, when $k \approx 1$, the monopolist would have to exclude almost all buyers in order to induce rationing at the public option, which entails the monopolist earning vanishing profits. In contrast, simply posting a price of $v^*(1-\theta)+\rho \approx \rho$ allows the monopolist to sell to almost all buyers at a sufficiently high enough price so that its profits are bounded away from zero. In the limit, as $k, \theta \to 1$, the mixed market resembles one of a duopoly market where firms engage in a Bertrand competition with one of the firms having zero marginal cost (the monopoly) and the other firm having a marginal cost of ρ (the public option).

Private Market as a Complement to Public Option In this section, I return to the baseline model where the public option supplies a good of the same quality as the monopolist for free. However, I consider the following alternate timing:

Timing: First, the monopolist selects a selling mechanism. Then, each buyer privately observes her valuation for the good and chooses whether to participate in the monopolist's mechanism or opt for the public option. For those who choose the public option, the good is allocated for free according to a random rationing rule. Buyers who fail to acquire the good from the public option may then turn to the monopolist, allowing buyers to "top up" their demand via the private market. Finally, for buyers who initially chose the monopolist or came to rely on it after failing to get the good from the public option, the mechanism determines whether they receive the good and how much they pay. Importantly, I assume that the monopolist cannot condition the mechanism on whether a buyer first approached the public option.[9]

Notice that in this setting, it is a weakly dominant strategy for each buyer type to first approach the public option. Thus, the rationing probability at the public option is k, regardless of the mechanism chosen by the monopolist. This also implies that a mass $1-k$ of buyers in the market need to top up their demand through the monopolist. Furthermore, because the public option is equally accessible to all buyers, the type distribution of buyers who seek to top up their demand is F.

Consequently, the monopoly faces a standard screening problem, and the optimal mechanism is a posted price at the standard monopoly price v^M. Given a capacity $k \in (0,1)$, the monopolist's profit is given by

$$\mathcal{P}(k) \coloneqq (1-k) \cdot v^M \big(1 - F\big(v^M\big)\big),$$

and the surplus of a type-v consumer is given by

$$\mathcal{U}(v,k) = \begin{cases} k \cdot v & \text{if} \quad v < v^M \\ v - (1-k) \cdot v^M & \text{if} \quad v \geq v^M \end{cases}.$$

As is clear from these expressions, a capacity expansion always lowers the monopoly profits and increases the surplus of low-value buyers ($v < v^M$). This is similar to the comparative statics established in Proposition 3 and Proposition 4. Moreover, in contrast to Proposition 5, a capacity expansion in this setting always increases the surplus

[9] If the monopolist could condition the mechanism on whether a buyer first approached the public option, then it would optimally refuse to sell to all buyers who did not initially choose the monopolist. The outcome would then be the same as the baseline model.

of high-value buyers ($v \geq v^M$) without additional conditions on the market primitives.

Intuitively, since the monopolist can no longer induce varying demand at the public option through its mechanism, expanding the public option's capacity improves the rationing probability without giving rise to any offsetting strategic behavior from the monopolist. Therefore, all buyers are more likely to have their demand met through a free public option rather than an expensive private market.

Finally, notice that type-v's surplus in a market served only by a public option is $k \cdot v$. Hence, introducing a monopoly to a market initially served only by a capacity-constrained public option is beneficial only to high-value buyers. This stands in contrast to Proposition 6 in which almost all buyer types benefit.

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